Approximate Classical Trajectories and the Adiabatic Theory of the Stark Broadening of Neutral-Atom Lines

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At low temperatures the effect of back reaction of the neutral emitter on a perturbing electron may become noticeable and consequently deviations of the electron motion from the uniform one should be taken into account. In this note we discuss some consequences of an inclusion of the curvilinear trajectories on the impact Stark broadening theory in the low temperature limit. In particular we consider a proposed improvement of the commonly used definition of the collision time, in the case of the repulsive electron-excited neutral interaction.

The asymptotic polarization potential $-V_p = C_4/r^4$, which a slowly moving impact electron "feels", may be attractive or repulsive, depending on the sign of the quadratic Stark constant C_4 of the target. If the electron moves in an attractive polarization potential, created by an excited neutral atom, its fate is determined by the critical impact parameter (atomic units are used throughout) [1]

$$\varrho_c = (8\,|C_4|/v^2)^{1/4} \tag{1}$$

where v is the impact velocity. If the impact parameter of the incoming electron is smaller than ϱ_c, the electron "falls" into the atom. The influence of deviations from the straight-line trajectory due to an attractive interaction, at medium temperatures, has been discussed in detail by the authors [1, 2]. On the other hand, in the low temperature region ϱ_c becomes so large that practically all electrons important for the broadening mechanism fall into the emitter. The perturbative approach as well as the very concept of a classical trajectory become questionable in the close vicinity of the target and one must resort to a more refined theory which accounts for a number of short-range effects. The situation is the same in the case of the excited hydrogen atom dipole potential [3].

In the case of a repulsive potential, however, an introduction of a nonuniform motion of the perturbing electron eliminates the problem of the minimum impact parameter (see e.g. Baranger [4]), which is a crucial difficulty in the low temperature region. On the other hand, an adoption of curvilinear trajectories requires a redefinition of the usual estimate of the so called collision time, commonly defined as: $\tau = 2\varrho/v$.

We adopt for the range of influence of the emitter on a charged perturber: $R_0 = 1.123\,\varrho_D$, where ϱ_D is the Debye radius. Now, we define τ as the time spent inside the sphere of the radius R_0.

This quantity is (e.g. Goldstein [5])

$$\tau = 2\int_{R_{\min}}^{R_0} \mathrm{d}r\,[r^2 - v^2\varrho^2/r^2 - V(r)]^{-1/2} \tag{2}$$

and for $V(r) = V_p$ one obtains

$$\begin{aligned} R_{\min} &= \varrho_c/\sqrt{2}\,,\\ \tau &= \frac{2}{v}\left[X_1^2\,F(\varepsilon,s)/a - a\,E(\varepsilon,s) + \frac{1}{R_0}\sqrt{(R_0^2+X_2^2)(R_0^2-X_1^2)}\right],\\ a &= (1+1/\tilde{\varrho})^{1/4}, \quad \tilde{\varrho} = (\varrho/\varrho_c)^4\,,\\ s &= X_2/a\,,\\ X_{1,2}^2 &= \varrho^2(1 \pm \sqrt{1-1/\tilde{\varrho}})/2\,,\\ \varepsilon &= \arccos(X_1/R_0)\,, \end{aligned} \tag{3}$$

where ϱ_c is defined by (1) and F and E are elliptic integrals of the first and second kind respectively (Gradshtein and Ryzhik [6]).

The collision time obtained in such manner differs considerably from that widely used in estimating validity conditions of various approximations incorporated in the Stark broadening theories for the neutrals.

As an example of the present approach we present in Fig. 1 results for the HeI $(3^1P^0\text{-}2^1S)$, $\lambda = 5017$ Å (multiplet 4) line, which has a large and negative quadratic Stark constant for the upper state of the transition: $C_4(3^1P) = -5.275 \cdot 10^4$ a.u. Further, the Stark constant of the lower level: $C_4(2^1S) = 400.57$ a.u. is negligibly small compared with the first one and we therefore neglect broadening of the lower level. The line has also an advantage that the monopole-quadrupole interaction may be ignored,

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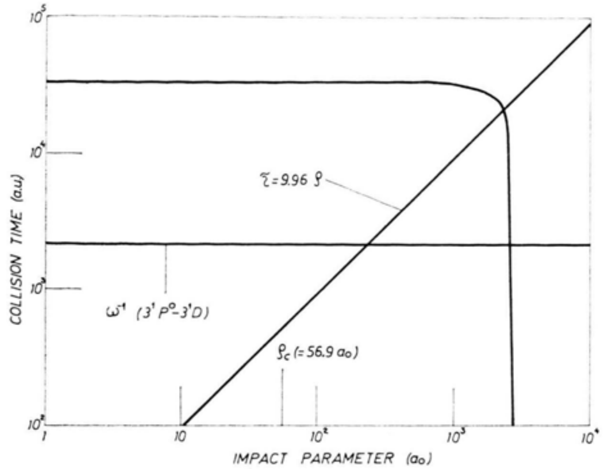

Fig. 1. Collision time for the electron-He(3^1P) scattering; $N_e = 10^{15}$ cm^{-3}, $T = 5000$ K.

since the quadrupole moment: $Q(3^1\mathrm{P}) = 73.2$ a.u. is by two orders of magnitude smaller than C_4. As can be seen from Fig. 1, the straight-path (high temperature) and curvilinear-path (low temperature) collision times are quite different for the plasma conditions chosen. This should be taken into account in considering conditions of the applicability of various approximations in the low-temperature limit. E.g., the usual adiabatic theory by Lindholm [7] and Foley [8] assumes that the collision time is much smaller than any of the characteristic times corresponding to virtual transitions to the perturbing levels. This means that for all significantly contributing ϱ-values one must have (e.g. van Regemorter [9]): $\tau \gg 1/\omega_{ij}$, where i refers to the initial (upper) level of the transition and j to a nearest perturbing level. In our case the latter is the 3^1D state and one has:

$$\omega^{-1}(3^1\mathrm{P}^0\text{-}3^1\mathrm{D}) \approx 2.1 \cdot 10^3 \text{ a.u.}$$

Hence, with the present definition of the collision time, for: $E < \Delta E(3^1\mathrm{P}^0\text{-}3^1\mathrm{D})$, the adiabatic approximation is justified for all relevant impact parameters, in variance with the usual definition, as can be seen from Figure 1. Here, E is the energy of the impact electron and ΔE is the energy difference from the nearest perturbing level.

The half-halfwidth and the shift of a line are given by the usual adiabatic formula (e.g. van Regemorter [9]), with v meaning the mean electron velocity for a Maxwellian distribution

$$w - i d = 2\pi N_e v \int_{\varrho_{\min}}^{\varrho_{\max}} (1 - e^{i\eta})\, \varrho \, d\varrho , \quad (4)$$

where the phase shift is calculated by

$$\eta = \int_{-\infty}^{+\infty} V_p(t)\, dt . \quad (5)$$

The use of v as a representative velocity in Eq. (4) instead of the average over the velocity distribution is justified at low temperatures, since the Maxwellian distribution is narrow there.

In our case: $\varrho_{\min} = 0$, $\varrho_{\max} = R_0$, and Eq. (5) yields

$$\eta(\varrho) = \xi(\tilde{\varrho})\, \eta^{(0)}(\varrho), \quad \eta^{(0)} = \pi C_4 / 2 v \varrho^3 , \quad (6)$$

where $\eta^{(0)}$ is the usual "straight-line phase shift" and the universal function: $\xi(\tilde{\varrho})$ is given by

$$\xi(\tilde{\varrho}) = \frac{16}{\pi} \varrho \sqrt{\beta} \left[E(\gamma) - \frac{\beta + 1}{2\beta} K(\gamma) \right],$$

$$\cdot \beta^2 = 1 + 1/\tilde{\varrho} , \; \gamma^2 = \frac{\beta - 1}{2\beta} , \quad (7)$$

where $\tilde{\varrho}$ is defined in (3) and $K(\gamma)$, $E(\gamma)$ are the complete elliptic integrals of the first and second kind, respectively [6].

In Fig. 2 we have plotted $\eta^{(0)}$ and the improved phase shift η as functions of the impact parameter. Evidently, $\eta^{(0)}$ deviates considerably from η for $\varrho < \varrho_c$.

The evaluation of the integral in (4) has been carried out numerically in the region: $\tilde{\varrho} < 3$ and the rest analytically, since for: $\tilde{\varrho} > 3$ one has: $\eta \approx \eta^{(0)} \ll 1$. For the line HeI($3^1\mathrm{P}^0$-$2^1$S), electron density: $N_e = 10^{15}$ cm^{-3} and temperature: $T =$

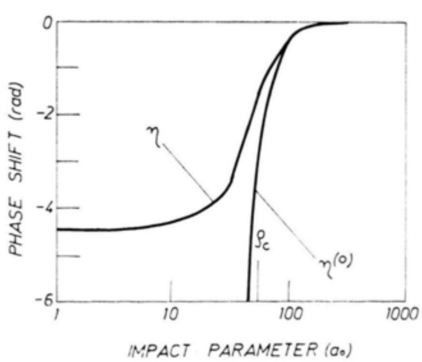

Fig. 2. e-He(3^1P) scattering phase shift, at $T = 5000$ K.

5000 K, one obtains for the half-halfwidth: $w = 0.0400$ Å, whereas the ordinary adiabatic theory calculations provide [10] the value: 0.0367 Å, which is to be compared with the semiclassical result [10]: 0.0378 Å. Thus, in the case investigated here the back reaction effect increases the adiabatic half-halfwidth by 9%, whereas the calculated value differs from the semiclassical result by approximately 6%.

It is of interest here to compare the behaviour of the shift to width ratio $-|d|/w$, calculated by the modified and by the ordinary adiabatic theories, at low temperatures. The results are presented in Fig. 3. One notices that the $|d|/w$ ratio increases as the temperature goes down, different from the prediction of the usual theory, which yields: $|d|/w = \sqrt{3}$. On the other hand, we note that the calculated behaviour is in accordance with earlier findings by the authors [2].

Though the low temperature region examined is of little laboratory relevance, it is of importance for a class of astrophysical regions, where both the temperature and the perturber density can be very low. On the other hand, as the plasma temperature rises, the role of the polarization back reaction effect diminishes, firstly, because the impact electrons become more energetic and deviate little from a straight-line path, and second because inelastic collisions become dominant [1].

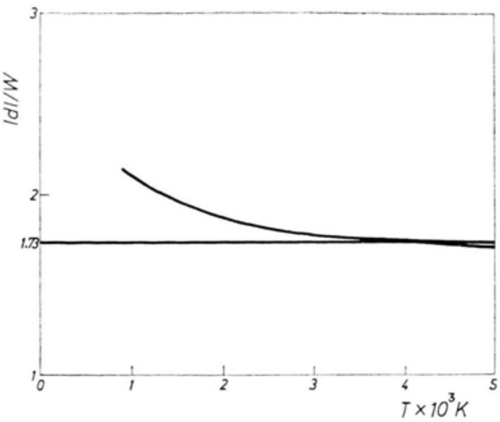

Fig. 3. Shift-to-half-halfwidth ratio for $HeI(3^1P^0\text{-}2^1S)$.

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